Topological Interference Management: Trade-off Between DoF and SIR for Cellular Systems

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***Abstract*—Topological interference management (TIM) allows studying the degrees of freedom (DoF) of partially connected linear interference communication networks, where the channel state information at the transmitters (CSIT) is restricted to the topology of the network, i.e., a knowledge of which interference links are weak and which are strong. In this paper, we consider TIM for an infinite downlink cellular network in the one-dimensional (1D) linear and the two-dimensional (2D) hexagonal models. We consider uniformly distributed users in each cellular cell, effectively creating a continuous distribution of users, aiming to study user classes based on different interference profiles rather than on actual individual users' positions. We also consider the construction of the TIM network topology by analyzing different interference thresholds. Unlike previous works, we use TIM at the user class level to find the system's DoF independent of the actual user position. Finally, after proposing a fractional coloring scheme that can achieve the optimal DoF solution, a trade-off between DoF and SIR is given.**

***Index Terms*—topological interference management, interference, topology, wireless systems, graph theory, cellular, degrees of freedom.**

I. Introduction

In spite of all recent advancements concerning wireless communications systems, interference remains one of the most critical "Achilles heels" of multi-user wireless systems, a problem that naturally arises due the broadcast nature of radio transmissions. Traditionally, interference has been dealt with the orthogonalization of resources, essentially avoiding interference [1]. However, the strict structure in the partition of resources is known to provide lackluster performance in many situations. Facing the problem of interference has become all the more important nowadays due the need to extract more and more capacity out of an ever shrinking pool of spectral resources.

Recent techniques, such as interference alignment (IA) [2], [3] for dealing with interference rely on channel state information at the transmitter (CSIT) or receiver (CSIR), cooperation between transmitters and receivers and other complicated signaling techniques. While such techniques perform very well in controlled settings, they have proven complicated to implement in real life systems and are not easily scalable to large networks. A clever way of understanding the impact of interference in large wireless systems was proposed in [4], where channel knowledge is not needed beyond the *topology* of the interference links. This technique, called topological interference management (TIM), bases its analysis on a classification of the interference links into relevant or non-relevant, giving rise to a topology. Based on the topology, graph analysis can be made to characterize the underlying network with respect to its degrees of freedom (DoF).

The analysis of cellular networks through TIM has been the subject of recent works. TIM itself as a technique has been introduced in [4], where the foundations of graph analysis for topological networks is laid out. Analysis for simple cellular network structures (linear, square and hexagonal, with users located in the border of the cells) is addressed therein, with their DoF characterized. The work in [5] extends [4] by adding multi-layer interference as well as asymmetrical user positioning along the border of the cell. The work in [6] shows that an orthogonal resource allocation scheme achieves the sum DoF for linear one dimensional (1D) convex networks. This work is later generalized in [7], where a linear 1D and two-dimensional (2D) cellular networks are studied through TIM, and it is shown that, if the resulting TIM network topology is chordal, then an orthogonal resource sharing scheme based on fractional coloring is sufficient to achieve optimal DoF performance. The authors show that one-dimensional convex TIM network topologies are chordal bipartite topologies. A bipartite topology graph is called a chordal bipartite topology graph if and only if any cycle of length 6 or more in this graph has at least one chord. A closed loop of a set of vertices and edges is called a cycle, and the chord is an edge connects two non-adjacent vertices of a cycle.

As stated before, defining a topology is crucial to perform TIM analysis. However, different interference thresholds will yield different topologies. As a matter of fact, even at a given interference threshold, different classifications of interference links may be considered, changing the topology accordingly. In this work, we will focus on defining a unique topology for a wireless network at a given interference threshold. As such the contributions of this work are as follows. First, unlike [4]–[7], where a discrete distribution of users is considered, we adopt a uniform linear distribution of users, where users that share the same interference profile can be grouped into a *user class*. We then apply TIM at the user class level to extract DoF information for the system independent of the actual user position. This approach is useful to characterize the average performance of the network. Second, we consider the effect of the interference threshold on the topology and the DoF of the system. By varying the interference threshold, an interesting trade-off arises: either we have a loosely connected graph, hence more DoF, but at the same time a higher interference since nodes contend more to access the medium, or a densely connected graph, hence less DoF, but at the same time a lower interference since nodes will choose a more conservative access to the medium. In this work we look for the best operating point in terms of interference threshold.

The remainder of this paper is organized as follows. Section II describes the infinite 1D (linear) and the infinite 2D (hexagonal) downlink cellular models studied in this work, along with the uniform user distribution adopted in the analysis. Section III presents the different topologies and their link to the interference threshold parameter. Additionally, this section provides the corresponding DoF analysis. Section IV presents the signal to interference ratio (SIR) analysis as well as the results of the DoF-SIR tradeoff study. Finally, section V provides final remarks of this work.

II. System Model

In this work, we consider two distinct downlink cellular models, namely, infinite 1D and infinite 2D downlink cellular networks, whose characteristics are now presented.

A. Infinite 1D Downlink Linear Cellular Network Model

For the infinite 1D downlink linear cellular network, as shown in Fig. 1, cells of coverage region of length L are placed uniformly along a straight line. We consider one base station (BS) in each cell, with total transmit power P, located at the center of the cell. Each BS has M messages to be sent to its own M mobile users, and each user receives a unique message from its home BS. A total of M users are distributed uniformly in each cell coverage region, with M chosen sufficiently large to be represented by a continuous linear distribution of users in the cells. Such a continuum representation is appropriate to perform a long-term resource allocation. To simplify the analysis, only path loss is considered in the channel gain. It is worth saying that TIM relying on a reduced feedback, cannot manage fading at the BSs. However, the impact of fading could be introduced in the model by replacing the classical rate-SIR relation given in (17), by a modified version such as $R = \beta\eta \cdot \log_2(1 + \Xi\mathrm{SIR}^{(w)})$ following [8], where β and Ξ are the spectral efficiency and the SIR efficiency respectively for a given system in some specific conditions. The channel gain between user m and BS i, is

$$\gamma_{i,m} = \frac{\rho}{\delta_{i,m}^{\alpha}}, \tag{1}$$

where $\delta_{i,m}$ is the distance between user m and BS i, α is the path loss factor, and ρ is the reference path loss for $\delta_{i,m} = 1$ meter. Also, in the subsequent analysis, the interference perceived by any user in the network, is as shown in Fig. 1, considered to come from its home BS (when it's serving other users in the home cell) and from all other BSs up to the second layer of interference, i.e., from the BSs in its adjacent cells and the first BSs following the adjacent cells. Hence, the distance above which we consider no link, i.e., $\gamma_{i,m} = 0$, for the interfering channels is lower bounded by $5L/2$.

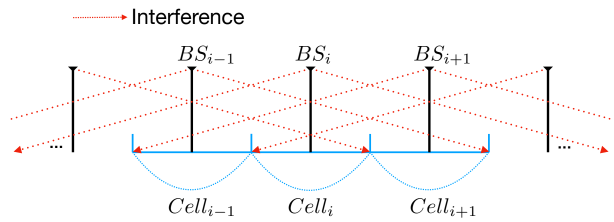

Fig. 1. Infinite downlink linear network model.

B. Infinite 2D Downlink Hexagonal Cellular Network Model

The second model considered in this work is an infinite 2D downlink hexagonal cellular network where the cells of radius R are placed according to an hexagonal grid pattern, with the same area A_{T}. We consider that each BS is equipped with an omni-directional antenna located at the center of the hexagonal cell with a total transmit power P. Similar to the 1D case, each BS has M messages to be sent for its own M mobile users, and each user receives a unique message from its home BS. Again, a large number M of users is distributed uniformly in the hexagonal cell area for all cells, and can be approximated by a spatial continuum of users. The channel follows the same considerations as for the 1D case in (1). We consider interference perceived by any user to come from its home BS (when it's serving other users in the home cell) and from only the first layer of interference, i.e., from the six BSs adjacent to its home cell, in its first surrounding layer given the hexagonal layout.

III. Network Topology and DoF Analysis

As introduced in [4], a topology must be determined to allow the calculation of the DoF via graph analysis. The path loss only model used herein allows a simple distance based interference connectivity threshold to be used for our TIM network topology construction. Therefore, the interference connectivity threshold is defined according to the interference distance D: significant interference links (i.e., connected interference links in the TIM representation) are only those such that $\delta_{i,m} \leq D$. In other words, in the TIM network topology, the interference links between nodes separated by a distance greater than D are considered disconnected (the corresponding channel coefficient is zero in the TIM representation), and the interference links between nodes separated by a distance D or less are considered connected (the corresponding channel coefficient is set to one in the TIM representation).

A. Infinite 1D Downlink Linear Cellular Network Model

Let us start with the infinite 1D downlink linear network, for which the corresponding TIM network topology is shown in Fig. 2. In our case, the interference connectivity threshold is bounded by $D < 5L/2$ because of the considered path loss and interference models from section II-A.

Each user m gets two kinds of links: the desired link, from its home BS, and significant interference link, from each BS i where $\delta_{i,m} \leq D$. The resulting network topology is then

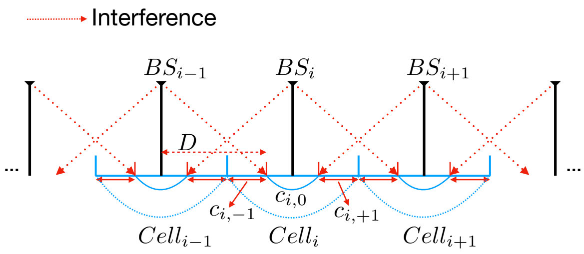

Fig. 2. Infinite linear network topology.

convex, as defined in [6]. This means that given any user m whose connectivity link to a BS i is guaranteed, then any user node $n \neq m$ whose distance $\delta_{i,n} \leq \delta_{i,m}$ must also be connected to the same BS i. Then, the 1D network topology is chordal bipartite, since all 1D convex network topologies are chordal bipartite [7].

We consider the most interesting case when $L/2 \leq D \leq L$. In this case, we can divide each cell i into three regions, as shown in Fig. 2: a cell center region $c_{i,0}$ and two cell edge regions $c_{i,-1}$ and $c_{i,+1}$. The length of the cell edge regions $c_{i,-1}$ and $c_{i,+1}$ is pL $\forall i$, and the length of the cell center region $c_{i,0}$ is $(1-2p)L$ $\forall i$, where $p \leq 0.5$ is the interference design parameter defined as

$$p = \frac{D - \frac{L}{2}}{L}. \tag{2}$$

Each user in cell region $c_{i,j}$, $j \in \{-1, 0, +1\}$, shares the same interference profile, therefore, a cell region defines a class of users sharing the same interference. The set of users in region $c_{i,0}$ is denoted by $U_{i,0}$, where these users see interference only from their home BS i, and it's defined as

$$U_{i,0} = \{\text{user } m \in \text{cell } i : \delta_{i-1,m} > D, \delta_{i+1,m} > D\}. \tag{3}$$

The set of users in region $c_{i,-1}$ is denoted by $U_{i,-1}$, where these users see interference only from their home BS i and from the BS $i-1$, and it's defined as

$$U_{i,-1} = \{\text{user } m \in \text{cell } i : \delta_{i-1,m} \leq D, \delta_{i+1,m} > D\}. \tag{4}$$

Finally, the set of users in region $c_{i,+1}$ is denoted by $U_{i,+1}$, where these users see interference only from their home BS i and from the BS $i+1$, and it's defined as

$$U_{i,+1} = \{\text{user } m \in \text{cell } i : \delta_{i-1,m} > D, \delta_{i+1,m} \leq D\}. \tag{5}$$

To study the DoF for this topology, we must first state some useful definitions that will be used in the following.

Definition 1 (Message Conflict Graph)**.** *The message conflict graph is an undirected binary graph, where each vertex represents a message W desired by user m and an edge exists between two vertices (messages) if the BS of one message interferes in the TIM representation with the user of the other message.*

Definition 2 (User Class Based Conflict Graph)**.** *The user class based conflict graph is an undirected binary graph, where each vertex represents a set of users $U_{i,j}$ belonging to the same class and located in $c_{i,j}$ and an edge exists between two vertices (two set of users) if the BS of one set of users interferes in the TIM representation with the other set of users.*

Remark 1. *The users who belong to the same class of users are represented by a unique vertex in the user class based conflict graph. The messages desired by these users form a clique in the message conflict graph. A clique in the message conflict graph is set of vertices (messages) such that any two are connected by an edge.*

The user class based conflict graph of our TIM network topology is presented in Fig. 3.

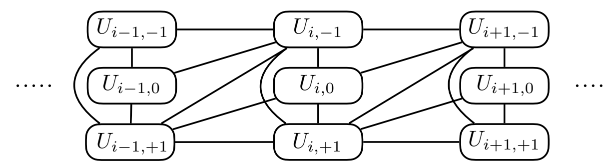

Fig. 3. The user class based conflict graph of the 1D topology in Fig. 2.

Definition 3 (DoF Region)**.** *The DoF region is the closure of all the achievable messages' DoFs.*

Remark 2. *Since each user in the network desires a unique message then the achievable DoF for a user is the achievable DoF of the message desired by this user.*

Definition 4 (Symmetric DoF)**.** *The symmetric DoF is the largest DoF inside the DoF region that can be achieved simultaneously by all messages (users) in the network.*

Remark 3 (Symmetric DoF in the Conflict Graph)**.** *In the message conflict graph, the symmetric DoF is the largest DoF value that can be achieved simultaneously by all nodes. This corresponds to a weighted symmetric DoF in the user class based conflict graph, in which each node in the user class based conflict graph achieves the symmetric DoF value weighted by its corresponding clique size in message conflict graph.*

Now let us proceed to the DoF solution of the 1D linear TIM problem defined in this section. The DoF solution is per cell and is based on guaranteeing the symmetric DoF for all users in the 1D linear network topology. The following theorem presents the DoF solution.

Theorem 1. *For the 1D linear cellular network topology with the interference design parameter p, the information theoretic DoF solution when all the users achieve the symmetric DoF value is*

$$\eta = \frac{1}{1+p}, \tag{6}$$

where η is the DoF value per cell.

Proof. For chordal bipartite TIM network topologies, the DoF region of the TIM problem is characterized through the cliques of the message conflict graph as presented in [7]. Based on this and using the user class based conflict graph shown in Fig. 3, the DoF region of our TIM problem is characterized by all the DoF solutions that satisfy

$$\begin{cases} d_{i-1,-1} + d_{i-1,0} + d_{i-1,+1} + d_{i,-1} \leq 1, & \forall i \\ d_{i-1,+1} + d_{i,-1} + d_{i,0} + d_{i,+1} \leq 1, & \forall i, \end{cases} \tag{7}$$

where $d_{i,j}$, $j \in \{-1, 0, +1\}$, is the sum of the DoFs of the messages desired by the set of users $U_{i,j}$.

Knowing that a large number M of users are distributed uniformly and continuously inside each linear cell and since we are targeting the symmetric DoF for all users, then each cell region's DoF will be proportional to the size (length in 1D) of the cell region. We can then state that

$$d_{i,j} = p\eta, \ \forall i \text{ and } \forall j \in \{-1, +1\}, \tag{8}$$

and

$$d_{i,0} = (1 - 2p)\eta, \ \forall i, \tag{9}$$

where $\eta = M d_{\text{sym}}$ and d_{sym} is the symmetric user DoF.

Based on (7), (8) and (9), we get

$$3p\eta + (1 - 2p)\eta = 1, \tag{10}$$

thus completing the optimal DoF solution in (6).

□

It is clear from (6), that as the interference design parameter p increases, the TIM network topology becomes more connected (i.e., more interference links are considered to be significant in the 1D linear topology) and the DoF per cell decreases.

B. Infinite 2D Downlink Hexagonal Cellular Network Model

Now, let us address the case of the infinite 2D downlink hexagonal cellular network. A possible corresponding TIM network topology is depicted in Fig. 4. This network topology is based on the threshold of interference connectivity which will be defined in our model as the interference distance D. We consider in this paper the case when $R \leq D \leq 3R/2$. It follows that each cell i can be divided into 13 cell regions, a cell center region $c_{i,0}$ and 12 cell edge regions $c_{i,j}$ where $j \in \{1, 2, \ldots, 12\}$.

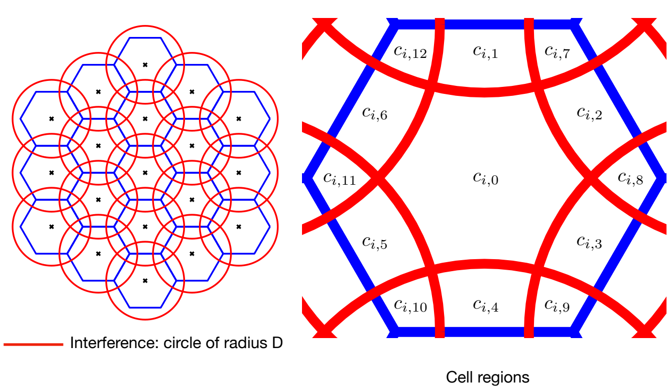

Fig. 4. Infinite hexagonal cellular network topology.

The set of users in cell i in the cell center region $c_{i,0}$ is denoted by $U_{i,0}$. These users see interference only from their home BS. The set of users in cell i in the cell edge region $c_{i,j}$ where $j \in \{1, 2, \ldots, 6\}$ is denoted by $U_{i,j}$ where $j \in \{1, 2, \ldots, 6\}$. These users see interference only from their home BS and from the BS in their unique adjacent cell. Then, the set of users in cell i in the cell edge region $c_{i,j}$ where $j \in \{7, 8, \ldots, 12\}$ is denoted by $U_{i,j}$ where $j \in \{7, 8, \ldots, 12\}$. These users see interference only from their home BS and from the BSs in their two adjacent cells.

The user class based conflict graph of our TIM network topology is presented in Fig. 5. The graph construction is based on the same properties as the user class based conflict graph for the 1D linear model, however in order to aid in readability, some simplifications were made: 1) the edges between users classes belonging to the same cell are omitted; 2) an edge connecting a given user class to a whole cell aggregates all edges connecting this user class to all user classes in that cell.

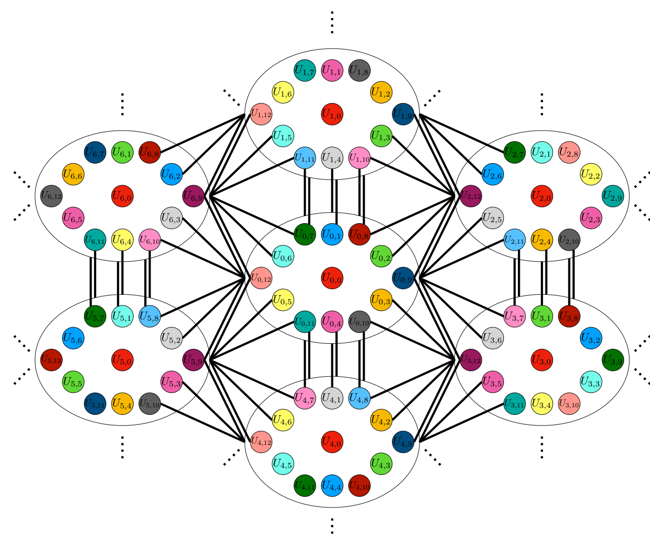

Fig. 5. The user class based conflict graph of the 2D hexagonal topology in Fig. 4. The 17 different colors shown in this figure correspond to the fractional coloring scheme solution in section IV.

Through the same analysis as in section III-A, and using the conflict graph of Fig. 5, the following theorem presents the DoF solution of the 2D hexagonal TIM problem.

Theorem 2. *For the 2D hexagonal cellular network topology with the interference connectivity threshold D, the DoF solution when all the users achieve the symmetric DoF value is*

$$\eta = \frac{1}{1 + \dfrac{A_1 + A_2}{A_T}}, \tag{11}$$

where η is the DoF value per cell, and A_T, A_1 and A_2 are defined, using [9], as follows

$$A_T = \frac{3\sqrt{3}R^2}{2}, \tag{12}$$

$$A_1 = \frac{1}{6}\pi D^2 - R^2\frac{\sqrt{3}}{4}, \tag{13}$$

$$A_2 = \frac{\frac{\sqrt{3}}{4}b^2 + 3\left[D^2 \arcsin\left(\frac{b}{2D}\right) - \frac{b}{4}\sqrt{4D^2 - b^2}\right]}{3}, \tag{14}$$

where b *is defined as*

$$b = \sqrt{3D^2 - \frac{3R^2}{2} - 3R\sqrt{D^2 - \frac{3R^2}{4}}}. \tag{15}$$

Proof. We will restrain from providing the whole proof here for brevity. The proof can be found similarly to the proof of Theorem 1, considering that the DoF for each cell region is proportional to the cell region size (area). □

Figure 6 shows the DoF per cell in the 2D hexagonal network topology as a function of the interference distance D. Based on the DoF solution in Theorem 2 and from Fig. 6,

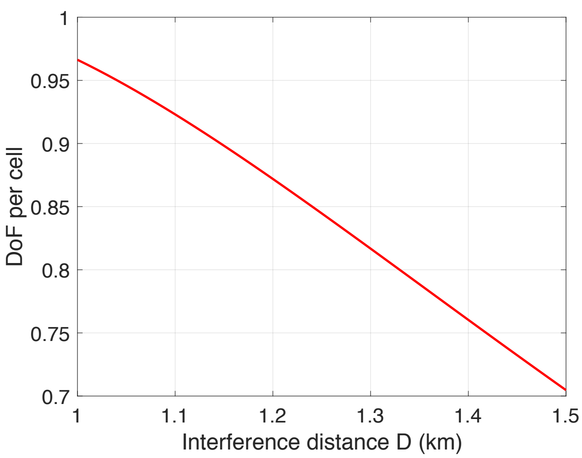

Fig. 6. DoF per cell as function of interference distance D.

it's clear that as the interference distance D increases, the TIM network topology becomes more connected (i.e., more interference links are considered to be significant in the 2D hexagonal topology) and the DoF per cell decreases.

IV. Trade-off Analysis Between DoF and SIR

Determining an optimal interference threshold is an important issue for TIM due to the trade-off between DoF and SIR. In this section, a fractional coloring scheme [10]–[12], achieving the optimal DoF solution for both 1D and 2D models is presented. Then, a SIR analysis, based on our proposed model and fractional coloring scheme, is presented. Finally, we show the trade-off between DoF and SIR.

A. Infinite 1D Downlink Linear Cellular Network Model

In [7], the authors show that if a TIM network topology is chordal bipartite, like the network topology considered herein, then an orthogonal access scheme, such as fractional coloring, is sufficient to achieve the DoF region of the corresponding TIM problem. We propose a fractional coloring scheme that achieves the optimal DoF solution of (6), pictured in Fig. 7.

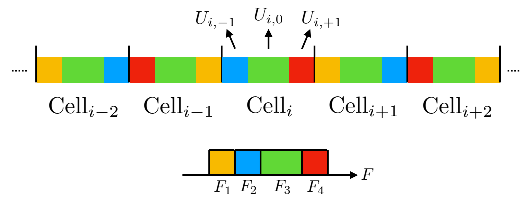

Fig. 7. Fractional coloring scheme solution for the linear model.

This fractional coloring scheme is designed such that the different frequency bands F_1, F_2, F_3, and F_4 are allocated for the different set of users $U_{i,j}$, $\forall i$, $\forall j \in \{-1, 0, +1\}$, in such a way that any two set of users that have the same allocated frequency band do not interfere in the TIM scenario (no edge between them in the conflict graph).

The SIR for any user m in cell i, using the proposed fractional coloring scheme, is

$$\text{SIR}_m = \frac{\gamma_{i,m}}{\sum_{j \in I} \gamma_{i+j,m}}, \tag{16}$$

where $I = I_0 = \{-2, -1, +1, +2\}$ if $m \in U_{i,0}$, $I = I_{-1} = \{-2, +1\}$ if $m \in U_{i,-1}$, and $I = I_{+1} = \{-1, +2\}$ if $m \in U_{i,+1}$.

Our objective is to find the cell rate when all users in the network have the same achievable symmetric user rate. This achievable cell rate is defined as follows

$$R = \eta \cdot \log_2\left(1 + \text{SIR}^{(w)}\right), \tag{17}$$

where R is the cell rate, η is as in (6) (multiplicative factor), and $\text{SIR}^{(w)}$ is the worst user SIR (log factor), determined by searching, in any cell i, for the worst user SIR in the cell center region and the worst user SIR in the cell edge regions. The worst user SIR in the cell center region $c_{i,0}$, denoted by $\text{SIR}^{(w)}_{i,\text{c}}$, is the SIR of user $m \in U_{i,0}$ where $\delta_{i,m} = \frac{L}{2} - pL$. The worst user SIR in the cell edge regions $c_{i,-1}$ and $c_{i,+1}$, denoted by $\text{SIR}^{(w)}_{i,\text{e}}$, is the SIR of user $m \in U_{i,-1}$ or user $m \in U_{i,+1}$ where $\delta_{i,m} = \frac{L}{2}$. The worst user SIR $\text{SIR}^{(w)}$ at a given p is

$$\text{SIR}^{(w)} = \min\left(\text{SIR}^{(w)}_{i,\text{c}}, \text{SIR}^{(w)}_{i,\text{e}}\right). \tag{18}$$

The achievable cell rate when all users in the network have the same achievable symmetric user rate, is shown in Fig. 8 for path loss factor $\alpha = 2$, as a function of the interference design parameter p. The trade-off between DoF and SIR, seen in Fig. 8, is as follows. On the one hand, if the interference connectivity threshold is too high (high p) then the topology becomes more connected (low DoF), and thus fewer opportunities to exploit TIM. Also, in this case with high SIR level, the cell rate will be low due to the low DoF at high p. On the other hand, if the interference connectivity threshold is too low (low p) then the topology becomes less connected (high DoF), and hence, plenty of opportunities to exploit TIM will arise. However, the SIR itself will suffer yielding low

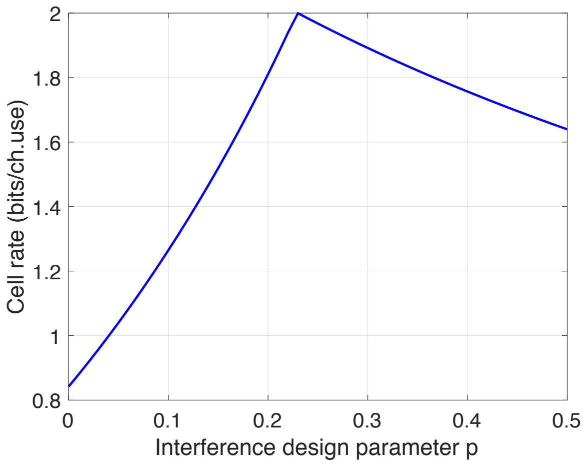

Fig. 8. Cell Rate as function of the interference design parameter p.

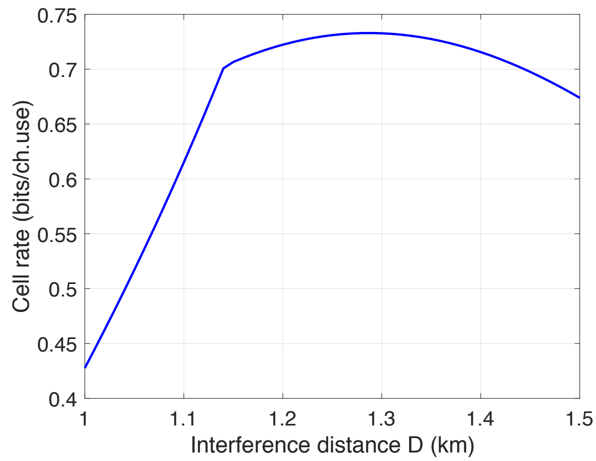

Fig. 9. Cell Rate as function of interference distance D.

cell rates. As a matter of fact, this results highlights that a topological feedback may be sufficient in a network to obtain good performance, but the choice of the interference threshold is of utmost importance.

In our scenario the optimal choice of the interference threshold (optimal interference design parameter p^*) happens when

$$\mathrm{SIR}_{i,\mathrm{c}}^{(w)} = \mathrm{SIR}_{i,\mathrm{e}}^{(w)}, \tag{19}$$

where $p = p^*$ fulfills (19), represented as the highest point of the curve in Fig. 8, with p^* being approximately 0.23.

B. Infinite 2D Downlink Hexagonal Cellular Network Model

For the 2D model, the fractional coloring scheme that can achieve the optimal DoF solution in (11), is pictured in Fig. 5. This fractional coloring scheme is designed such that 17 different frequency bands are allocated for the different set of users $U_{i,j}$, $\forall i$, $\forall j \in \{0, 1, \ldots, 12\}$, in which any two set of users that have the same allocated frequency band do not interfere in the TIM network model (no edge between them in the conflict graph). The achievable cell rate is the same as for the 1D case in (17), but with η being the DoF per cell as defined in (11).

The achievable cell rate when all users in the network have the same achievable symmetric user rate, is shown in Fig. 9 for path loss factor $\alpha = 2$, as a function of the interference distance D, where the cell radius R is chosen to be 1 km. The horizontal range in Fig. 9 is between 1 and 1.5 due to the range of D given in section III-B. The conclusions taken for the 2D case are the same as for the 1D case, with the optimal interference parameter D^* being approximately 1.29 km.

V. Conclusion

The TIM problem for a downlink infinite cellular network is considered in the 1D (linear) and the 2D (hexagonal) models. The users are distributed uniformly in each network cell. In order to study user classes based on different interference profiles, a continuous distribution of users is considered. The TIM network topology is then built upon the interference distance threshold and optimal DoF solutions are computed accordingly. A fractional coloring scheme achieving the optimal DoF solution is proposed. We then show the trade-off between DoF and SIR in the network performance, relying on the interference threshold parameter. This latter result shows that the efficiency of a TIM approach is strongly impacted by an appropriate selection of the interference threshold parameter.

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